Design of a Reinforced Concrete Steel Arch Bridge

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Design of a Reinforced

CONCRETE STEEL ARCH BRIDGE

A Thesis presented by

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to the

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of the

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for the Degree

of

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Having completed the prescribed course of study in

Civil Engineering.

Chicago, June 1905.

H. M. Raymond
Dean of Engineering Studies.

L. C. Monin
Dean of Cultural Studies.

Alfred E. Phillips
Prof. Civil Engineering

The Design of a Reinforced Concrete Arch Bridge.

Data:--

The design of this bridge was made to cover an actual case, with the exception of its length. In the actual case five spans of 50 ft. each and two of 40 ft. would be necessary, while for the purpose of this design, one span of 50 ft. and 2 of 40 ft., were chosen. The piers and abutments are set on a solid limestone rock foundation. The Elevation of the crown of the road at the ends is 19.15 ft.; the bottom of river is 4.5 ft., and practically level. The assumed elevation of springing line is 8.5 ft. Width of road was assumed as 24 ft.; together with two sidewalks of 6 ft. each, gives a total width of Roadway of 36 ft. The Roadway was given a grade of 5% from the ends towards the center.

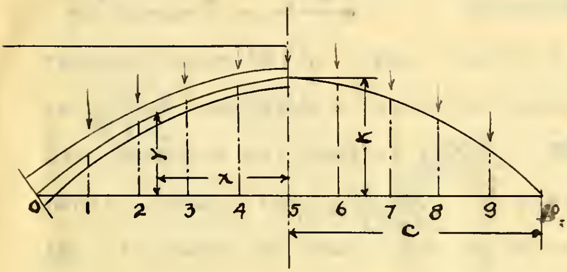
In the following discussion all figures will refer to the 50 ft. span arch. The design of this arch will be given in detail, while merely the figures for the 40 ft. span will be given.

Vertical Dead Load:--

The weight of concrete was taken as 130 # per cu. ft., the earth filling as 100 # per cu. ft. The factor of safety for the dead load is four (4).

The line of stress of the arch was assumed to be of parabolic form. For the 50 ft. arch, a rise of 8 ft. was assumed. From existing structures and plans made by the St. Louis Expanded Metal Company, a rough plan of the arch ring was drawn, thickness

of crown as 18 in., of haunches 24 in. By scale, the actual dead load of the arch was computed. For figuring the stresses, the loads were assumed as applied at 10 panel points numbered as shown in sketch. In table "concrete area" is the area of the arch ring at the panel point. This multiplied by the 130, gives the weight of the concrete applied at that point, the arch ring being assumed to be one foot wide. Likewise the area of filling was scaled, and this multiplied by 100 gives the weight of filling. The sum of these two X the factor of safety of 4, gives the Dead panel load for that point.



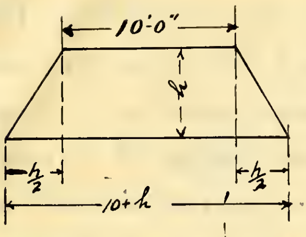
Vertical Live Load:--

A concentrated load of a 20 ton Road Roller, on 2 arches, 5 ft. long and 10 ft. between centers, was used to determine an equivalent uniformly distributed live load.

This load of 40,000 lbs. can be considered as distributed over 20 ft. of roadway or equals 2000 lbs. per lineal foot of Roadway. The axles being 5 ft. long, the broad tires of the roadroller would further distribute the load over about 10 ft. of width, giving a load of 200 # per lineal feet of arch one foot wide.

The actual pressure on the arch rib is much less than at

the crown i.e. the earth filling further distributes the Live Load. The angle of repose being more nearly vertical in all



probability than ordinary earth, in equilibrium, say 1/2 to 1, the 2000 # distributed over 10 Ft., at the sur face at a depth of h, below the surface would be distributed over $10 + h$. feet. The load per sq. foot of arch then is $\frac{2000 \#}{10+h}$ and with a factor of safety of eight (8) gives the Live Load per sq. foot as $\frac{16000}{10+h}$. The Live Load per panel point, panels being 5 ft., $= \frac{80,000}{10+h}$. 'h' was measured from the sketch and the Live Load for the different points calculated .(See table).

Horizontal Dead Load:--

According to Rankin's earthwork formulae, the horizontal intensity is to that of a vertical load as 1:3 when angle of surface $= 0^{\circ}$ and angle of repose $= 30^{\circ}$.

However the horizontal pressure acts on smaller surface than the corresponding vertical load i.e., if p = panel length, h = vertical distance between adjacent panel points. P = vertical load and h = horizontal load, $h = \frac{Ph}{3\,p}$. The Horizont al Dead Loads were accordingly calculated.

Horizontal Live Load:--

Was calculated in a similar manner.

Stresses:--

The stresses in the arch rib, due to bending moment,

thrust,and shear were figured according to Prof.Greene's method as given in his book,"Trusses and Arches"Part lll,Pages 60 to 62 & 116 to 119. The arch rib is considered to be of Parabolic shape with fixed ends. The actual tables for calculating the bending moments,thrusts and shear,were taken from Walter W. Colpritts' book on the "Calculation of the Stresses and Practical Design of Structures of Steel Concrete." The figures in the tables as given in the latter are merely those of Greene multiplied by twelve,to reduce the bending moment to inch pounds. It was not considered necessary to reproduce these tables as these constants can be found in text books and back numbers of the Engineering News.

The Actual figures however are given in the following tables:-

Bending Moment.Vertical Live Load Table

Horizontal " " "

Vertical Dead " "

Horizontal " " "

Thrust " " "

Shear " " "

Temperature;--

Again according to Prof. Greene,the Bending Moment at the crown due to a change of temperature =

$$\frac{15}{4} \times \frac{t e E I}{12 k}$$

If $t = 75^{\circ}$ F.,and e = Coeff.of expansion of concrete =.0000055 per degree F. E = Mod. of Elasticity of concrete = 3,000,000 pounds per square inch,and I = Moment of Inertia of Section at crown

thrust, and other more figures necessary, [illegible]'s method as given in his book, "[illegible] and [illegible]: Part [illegible], Pages 60 to [illegible] & 110 to 119. The arch [illegible] is considered [illegible] of Parabolic shape with fixed ends. The [illegible] tables for calculating the bending moments, thrusts and shears, were taken from Walter [illegible]. [illegible]' book on the "Calculation of the Stresses and Practical Design of Structures of [illegible] [illegible]." The figures in the tables as given in the [illegible] [illegible] [illegible] of Greene multiplied by twelve, to reduce the [illegible] [illegible] to inch pounds. It was not considered necessary to reproduce these tables as these constants can be found in [illegible] [illegible] [illegible] moment in the [illegible] [illegible] [illegible].

The [illegible] figures [illegible] [illegible] [illegible] in the following cases:-

Moving [illegible]. Vertical [illegible] [illegible] [illegible]

Horizontal " " "

Vertical [illegible] " "

Horizontal " " "

[illegible] " " "

[illegible] " " "

Temperature;--

[illegible] [illegible] to [illegible]. [illegible], the [illegible] Moment at the [illegible] due to a change in temperature =

$$\frac{15}{4} \cdot \frac{t\,e\,E\,I}{15\,K}$$

If $t = 75°$ F., and e = Coeff. of expansion of concrete = .0000055 per degree F. E = Mod. of Elasticity of concrete = [illegible] pounds per square inch, and I = moment of Inertia of section at crown.

then $M_t = 387 \frac{I}{K}$ $M_{uc} = .387\ k^{2}$

At the springing point, the bending moment is twice that at the crown or $774 \frac{I}{K}$

The Horizontal thrust under the same conditions $96 \frac{I}{K^2}$.
Shear due to a change of temperature was considered to be so small as to be negligible.

By multiplying the bending moment at the crown by the following factors, the bending moment at the respective panel point may be obtained.

Panel Points	Abut.	1	2	3	4	5
Factors	2	.92	.08	.52	.88	x1.0

These factors are obtained by assuming a uniformly distributed load as having the same effect as that due to a change of temperature. The bending moment at each panel point due to a uniform load, can be calculated from the tables in terms of the load, and then a ratio established between the moments at the various points and that at the crown.

It is necessary to have a value of the moment of inertia at the crown. Consider the formula for Bending Moment as deduced in Mechanics $M = \frac{FI}{y}$. Where F = tension or compression in outer fibre and y. is the distance of that fibre to the neutral axis, and I the moment of inertia then $I = \frac{My}{Fc}$

$I \frac{My}{Fc}$

Then finding the stress F_c due to the maximum moment, resulting from dead and live loads and calculating the distance

y.,by methods hereafter to be explained, I can be obtained, and consequently bending moment and thrust due to temperature.

In the design of sections Prof. Hatt's theory was adopted as being the most easily deduced and rational. The deduction of his formulae are given in Engineering News. Vol.47. P.170 and a brief outline of it follows.

Theory of the Strength of Beams of Reinforced Concrete

by Prof. W. Kendrick Hatt.

Engineering News. Vol. 47 P.170

An arch ring is considered as a beam in which each face may be in tension.

A. Steel - Modulus of Elasticity $= E_s = 30,000,000$ lbs.

Elastic Limit $= f = 30,000$ lbs.

B. Concrete in compression #,Mod. of Elasticity $E_c = 2,400,000$ lbs.

Compressive strength $= c = 2000$ lbs. = maximum

C Concrete in tension strength $= \frac{1}{10}$ of $c = 200$ lbs.

Computation:--

Material like concrete is not perfectly elastic. The stress-strain curve is not therefor ~~xxxxxx~~ a straight line up to the elastic limit and Hooke's Law cannot be assumed to hold.

The problem is to compute the ultimate or breaking load of a beam.

Assumptions:--The cross sections of the beam remain plane surfaces during flexure.

2. The applied forces are perpendicular to the neutral surface of the beam.

3. It is assumed that the pressure of the material surrounding any elementary fibre will not modify the effect of the stress on the fibre but that the latter will elongate or be compressed just as if it were under that load by itself in a testing machine.

4. There is no slipping between the faces of the wire and the surrounding concrete.

5. The elastic limit of the concrete is exceeded in both the compression and tension flanges.

6. There are no initial stresses due to shrinkage or expansion, of the concrete while setting.

The stress-strain diagram may be used to represent the Law of increase of Stress as we go from the neutral axis outward.

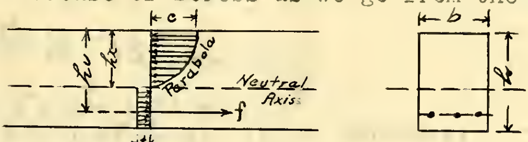


Fig. 3. illustrates the state of stress as adopted by Prof. Hatt.

Ordinarily there are given the following quantities Moduli of steel and concrete tensile and compressive, strength of concrete, size of beam and reinforcement with location of latter.

There are three conditions to be determined:

1st, the distance of the neutral axis from the upper face.

2nd, proportion of steel

3rd, moment of resistance

These can be determined by the algebraic statement of three facts; first, the total force of compression on compressive side equals the total force of tension on the tension side. Second, the extension of steel and the compression of concrete at the fibre will be to each other as the distance of these materials from the neutral axis. 3rd, the moment of external forces on one side of the section about the neutral axis = the moment of resisting stresses on the section about the same axis.

Prof. Hatt assumes the curve of compression to be a parabola and the tensional stress diagram a straight line paralell to the section.

H = depth of beam

b = width of beam

hx = distance of neutral axis from top of beam

F = area of concrete

F' = area of steel

$p = \frac{F'}{F} = \frac{1}{75}$ to $\frac{1}{100}$ generally

x, u & p are ratios.

E_c E c = Mod. of Elasticity of Concrete.

E_s E s = Mod. of Elasticity of Steel

f ~~F~~ = stress of steel

c ~~C~~ = compressive stress in outer fibre of concrete

t = tensional " " " " "

Area in compression = 2/3 c.h.x.

Total force of compression = 2/3 c.h.x.b.

Moment of compression about neutral axis = 2/3 c.h.x.b 5/8 h,x. ~~(2/3CHXB 5/8 HX)~~

Total force of tension on concrete = t,h,(1-x)b.

Moment of tension about the neutral axis = t,h (1-x)b,h $\frac{(1-x)}{2}$

Tension on steel = F'f = p h b f

Moment of this tension on steel about the neutral axis = p h b, fh (u-x)

From fact(1) previously referred to, we have 2/3 c h x b = t h (1-x) b + p h b f, or 2/3 c x = t (1-x) + p f (1)

From Fact 3.

$$M = 2/3\ c\ h\ x\ b,\ 5/8\ h\ x + t\ h\ (1-x)\ b\ h\ \frac{(1-x)}{2} + pbhfh\ (u-x)$$

$$M = b\ h^2 \left\{ t\ \frac{(1-x)^2}{2} + 5/12\ c\ x^2 + p\ f\ (u-x) \right\} \quad (2)$$

Let e_c = compression of fibres by compression and l_s = lengthening of fibres by tension of steel, then from definitions of E, $E_c = \frac{c}{l_e}$ and $E_s = \frac{f}{e_c}$ $\left\{ \text{or } E_s = \frac{f}{e_c} \quad E_c = \frac{c}{l_s} \right\}$

From fact (2)

$$\frac{e_c}{l_s} = \frac{h\ x}{h(u-x)} \quad \text{or} \quad \frac{c}{f} = \frac{E_c}{E_s} \frac{x}{u-x} \qquad (3)$$

Eliminate f from (1) & (3)

$$t(1-x) + p\ c\ \frac{E_s}{E_c} \times \frac{u-x}{x} = \frac{2}{3}\ cx. \qquad (4)$$

Having assumed c, t E_c & E_s as well as u and f, from equation (3) value of x can be computed. With this value of x, p can be computed from (4). Then with these values, having given your moment of external forces, the value of h can be found from (2).

By this method a table was made giving the value of M,x and p for different values of c.

In the computation u was assumed as 7/8. This gives a 2 inch covering for the steel in a 16 inch. beam. This value is a little low,but it allows for slight irregularities in placing the rods by the workmen. A rod may be bent slightly and if not straightened out may be at a greater distance from the face than 1 inch or 1 1/2 as ordinarily allowed.

Design of Section of the Crown.

The design of the sections at various panel points varies very little and as an example,the complete work for the section at the crown follows:

The vertical dead load moment at the crown = -170564 In. lbs. The horizontal dead load moment = -16728 inch.lbs. Live Loads,vertical,1-2-3-7-8-9 on the arch give a maximum negative bending moment of -194136 inch lbs. Horizontal Live Loads placed at these points give a moment of-12314 in.lbs.

Loads placed at 4-5-6 give a maximum positive moment of +89132 and horizontal loads placed at these points give a moment of -1284 inch lbs. Live loads producing positive moment give a total moment for the crown equal to -99444 inch.lbs. Live loads producing negative moment,give a total negative moment of 393742 inch.lbs. The negative moment of the dead load always overbalances the positive moment of the live load and consequently no positive moment at the crown.

The horizontal thrust due to the loads giving a moment of -393742 equals 63280 lbs.

Loads giving maximum moment usually also give the maximum thrust.

As stated before,it is necessary to design a temporary value for the section in order to find a value for the moment of inertia.

Assuming an allowed value of 1750 lbs. per sq. inch on the concrete to resist the bending moment from the designing table

$$M \text{ equals } 3488\, h^2$$

$$\text{or } h \text{ " } \sqrt{\frac{M}{3488}}$$

$$M \text{ equals } 393742$$

$$\text{" } h \text{ " } 10.6 \text{ ".}$$

and the area of the section equals 12" X 10.6" or 127.2 □"

The thrust per square inch on the section then equals

$\frac{63480}{1272}$ equals 500 #

The total allowed thrust for concrete equals 2000 # The direct thrust taking 500 # of this,the amt. left. i.e., 2000-500 or 1500 # can be utilized for bending moment. A new value of c. must therefor be chosen.

Let c equal 1500,then h equals $\sqrt{\frac{393742}{2686}}$ equals 12.2"

This gives an area of 12.2 X 12 equals 1464 sq. inch. The thrust then per square inch equals $\frac{63480}{146.4}$ equals 433 lbs. 1500 plus 433,equals 1933 lbs. total compression on concrete,well within the limit of 2000 #

For this value of c,x equals .336 h or 4.06

The moment of inertia equals $\frac{M \times y}{c}$ equals $\frac{393742 \times 4.06}{1500}$

equals 1068.

The moment at the crown due to temperature equals 387 $\frac{I}{K}$

equals 51700 " lbs.

The horizontal thrust equals $\frac{.096\ I}{K^2}$ equals 1602 lbs.

The total negative moment therefor equals -393742 + 51700 equals -445442. The minimum moment equals -99444 plus +51700 equals - 47744, since the moment due to temperature can be either negative or positive accordingly as there is a rise or fall in temperature, i.e., the moment must be added arithmetically to the moment due to the actual loads.

The total horizontal thrust equals 63280 plus 1602 equals 64882 lbs.

These are the final figures for the design of the section

Assumed c equals 1500 #

$$h = \sqrt{\frac{445442}{2682}} \text{ equals } 12.87''$$

Area of section equals 12 x 12.87 equals 154.24 sq. inches

$$\text{Pressure } = \frac{64882}{154.24} \text{ equals } 418. \text{ lbs.}$$

The total compression equals 1500 plus 418 # equals 1918 #

Interpolating for a new value of c.

2000-1918 equals 82. Let c equal 1500 plus 82 equals 1582 lbs. For this value of c, by interpolation, M equals 2953 h^2

$$h \text{ equals } \sqrt{\frac{445442}{2953}} \text{ equals } 12.28''$$

$$\text{Pressure equals } \frac{64882}{12.28 \times 12} \text{ equals } 437.5 \text{ lbs.}$$

and total compression equals 1582 plus 437.5 equals 2019.5 lbs.

This value of total compression is a little too large and a new calculation might be gone through but for all practical purposes h. may be assumed as 12.3" x, for c equals 1582, x is equal to .347 h or 4.27 ". Value of p. for c equals 1582 equals .00778 or the area of steel for the section equals .00778 x 12 x 12.28 equals 1.136 sq. inch.

There being no positive moment,this completes the design of the section for the crown.

An area of steel equals 1.136 sq. inches for a width of 12 equals practically 7/8 " round rods spared at 5 1/2 " center to center.

The sections at the other panel points are calculated in a similar manner.

When the values of x and h have been obtained,for both positive and negative bending moments,for every panel point,the values of h-x are laid off to scale above the parabola for negative moment and below the parabola for positive moment at the proper panel points. A smooth curve drawn through these points,one above and one below the parabola give the extradosal and intradosal lines of the arch. Under no circumstances may these lines go within the plotted points. At the haunches and even at the other points,~~xxxx~~ they are liable to fall very much outside of the points,when the area of steel may be decreased proportionally to its increased distance from the neutral axis,the parabola.

The area required for shear will be found to fall within the area required for bending moment and thrust and,consequently,no reinforcement is required for this. However to prevent any tendency of the steel bars on the lower side from straightening and consequently tearing out of the concrete when stretched,it is well to put in inclined bars,inclined about 45^{o},hooking the lower bars to the upper bars and thus preventing any such tendency. These bars will also take up any shearing stresses that may not have been prepared for.

[illegible]

of the section [illegible]

An area of [illegible] equals [illegible] .108 sq. inches for a width of 12 [illegible]
equals practically [illegible]
[illegible]

The [illegible]
a [illegible]

That the values of [illegible] have been obtained, [illegible]
positive and negative [illegible],
[illegible]
negative [illegible]
the [illegible] points. [illegible]
points, [illegible]
and [illegible]
these lines [illegible]. [illegible]
[illegible]
[illegible] decreased
proportionally [illegible]
[illegible]

The [illegible]
the bars required [illegible]
[illegible] reinforcement [illegible]
the tendency of [illegible]
[illegible]
[illegible]
the [illegible]
[illegible]. These bars will [illegible]
that [illegible]

Piers and abutments were designed to conform with the three following conditions.

1st: Area of base must be sufficient to give a unit pressure on both concrete and rock foundation, less than the crushing strength of curve.

2nd: Line of stress, due to eccentric loading must fall within the middle third of the base.

3rd: The angle of the resultant with the vertical, must be less than the angle of friction of the material.

The method used for first is obvious. To satisfy the second condition, the loading giving max. + and max. - M. at the abutments was considered since this loading gives the condition of maximum eccentricity and consequent instability. The eccentricity is equal to the bending moment divided by resultant of the vertical and horizontal components of the reaction due to the loading.

Piers and abutments were figured graphically and all steel used in them is merely for bending and temperature and not to resist stresses that may come into them due to the loading.

The Spandrel Walls were figured as horizontal beams between buttresses, the latter being figured as cantilevers. The same designing tables were used as those for the arch rib. All other wires in the structure are for the purpose of preventing surface cracks.

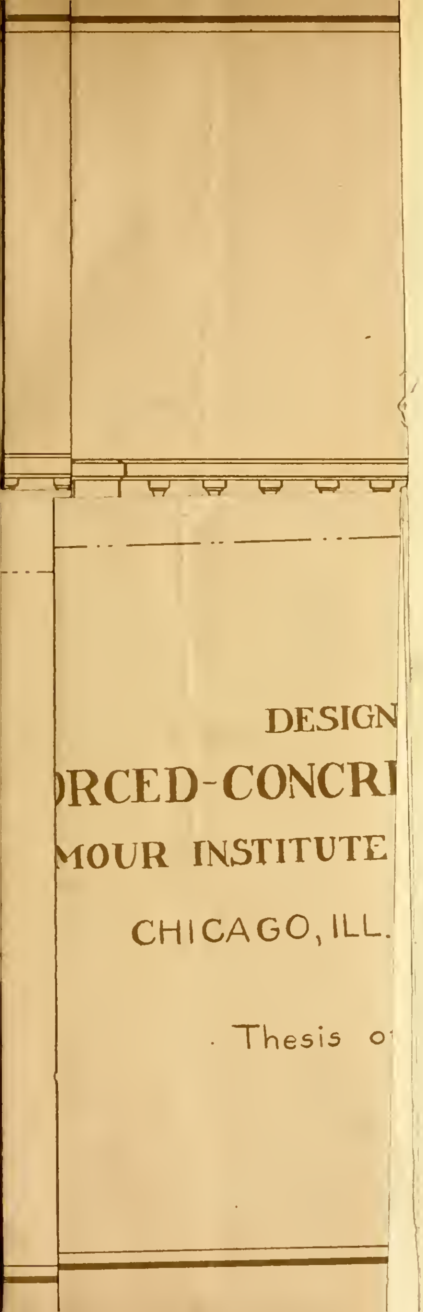
The railing and posts were copied from the Wabash R.R.Co's design of Forest Park Bridge at St.Louis.

In the design of Reinforced Concrete Arch many assumptions must be made. Prof. Greene makes assumptions in his theory of stresses in an arch. Prof Hatts makes assumptions in his theory of Remtance of Beams to flexure. Allowed stresses are assumed. large factors of safety or ignorance are used to take care of our lack of knowledge in regard to reinforced concrete. Notwithstanding all this until we have a fuller knowledge of Reinforced concrete (and this present Reinforced Concrete fad will bring it out), this method gives us a means of designing Concrete Arches economically, Arches that will hold the load for ~~that~~ which they are designed and yet have no more material in them than seems necessary.

John C. Penn

In the design of Reinforced Concrete beams and [illegible]

must be made. Prof. Turneaure makes experiments [illegible] theory of stresses in [illegible]. Prof. Marsh makes experiments [illegible] theory of Resistance of Beams to Flexure. Although theories are assumed, large factors of safety are employed [illegible] on account of our lack of knowledge in regard to reinforced concrete. Notwithstanding all this until a more definite knowledge of Reinforced Concrete (and this present Reinforced Concrete Age will bring it out), this method gives us a means of designing Concrete structures economically, Although [illegible] will hold the load for which they are designed and yet [illegible] no more material in [illegible] than is necessary.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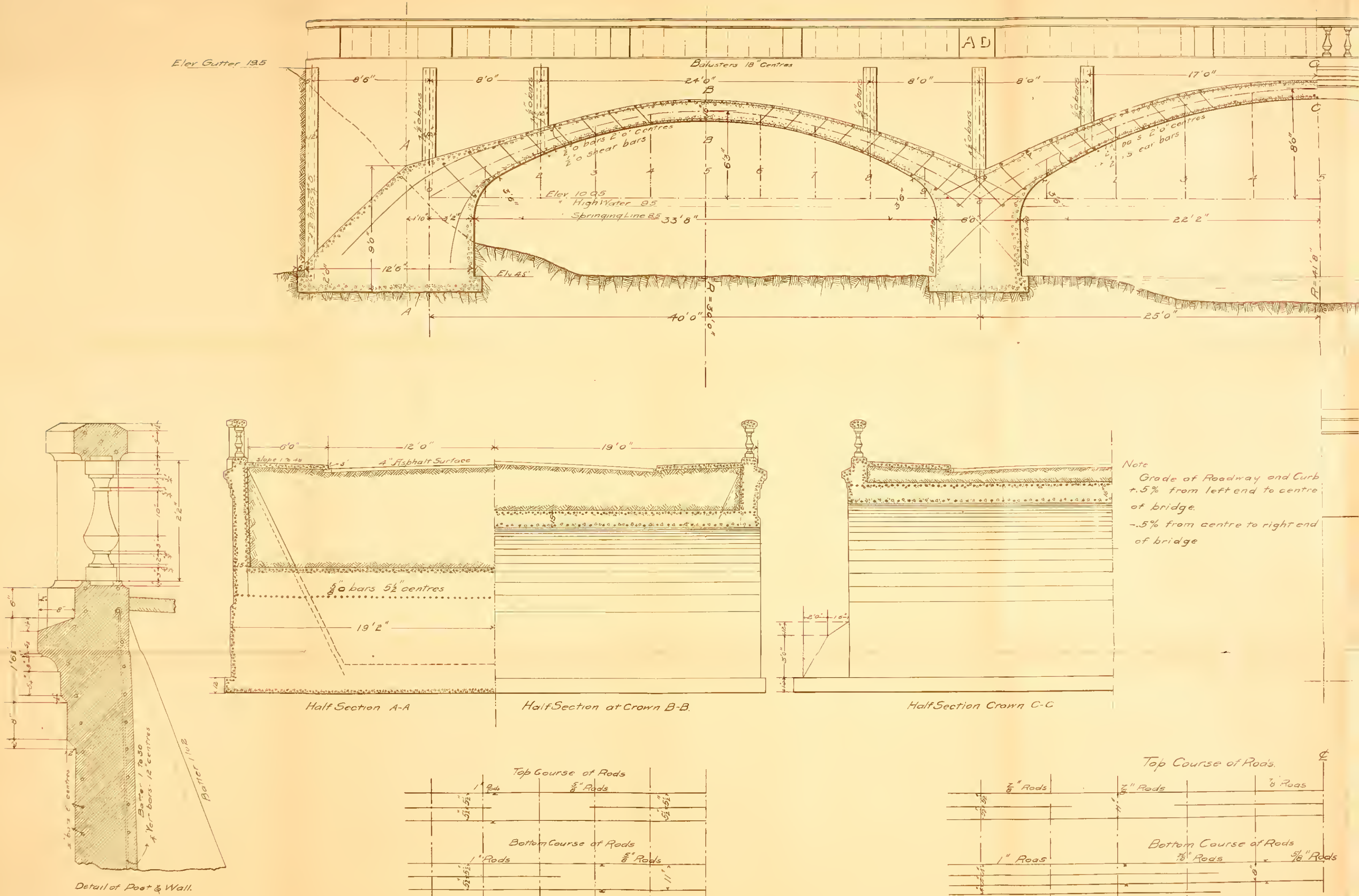
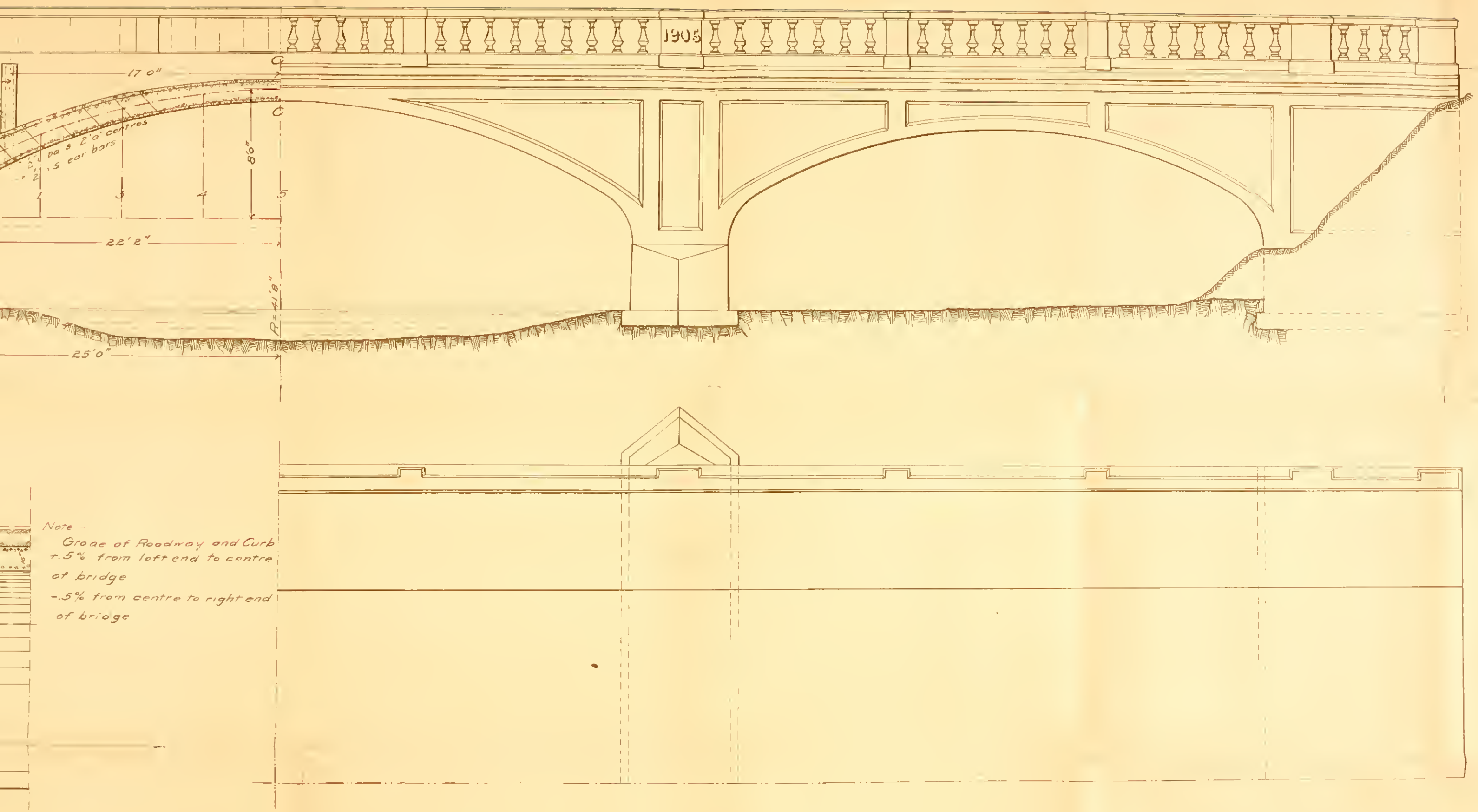
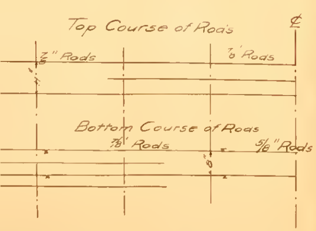

Elev. Gutter 19.5
Balusters 18" Centres
AD
8'6"
8'0"
24'0"
8'0"
8'0"
17'0"
Elev. 10.05
High Water 9.5
Springing Line 8.5
33'8"
22'2"
12'6"
Elv. 4.5
40'0"
25'0"
Half Section A-A
Half Section at Crown B-B.
Half Section Crown C-C
4" Asphalt Surface
19'2"
Note
Grade of Roadway and Curb +.5% from left end to centre of bridge.
-.5% from centre to right end of bridge
Detail of Post & Wall.
Batter 1 to 12
Top Course of Rods
Bottom Course of Rods
1" Rods
Top Course of Rods.
Bottom Course of Rods
1" Rods





DATA.

Three Spans 2- Length 33'8' Rise 7' 1½"
1- " 44'4" " 8'9"

Width of Roadway 24'0" 2 Sidewalks, @ 6'0"

Total width of Bridge 38'4"

LiveLoad - 20 Ton Road Roller, or 200 lbs pr sq ft.

DeadLoad - Filling, 100 lbs pr.cu ft Arch, 130 lbs pr cu.ft

Allowed Compression Concrete 2000 lbs pr. sq in

" Tension " 200 " " " "

" " Steel 30000 " " " "

Factors of Safety LL-8 DL-4

Scale.- ¼" = 1'0".

DESIGN OF

REINFORCED-CONCRETE ARCH BRIDGE

ARMOUR INSTITUTE OF TECHNOLOGY

CHICAGO, ILL. MARCH, 1905.

Thesis of { John C. Senn [?]
Shular Dean [?]

Table

Ordinates to Parabola

Panel point	0	1	2	3	4	5
Ordinate	0	288	512	6.72	7.68	8.0

$$y = h\left(1 - \frac{x^2}{c^2}\right)$$

y = Ordinate
h = mid " , or rise
x = absoissa
c = half-span

Designing Table for Bending Moments.

	c	M	hx	p
1	1000	$1161h^2$	.257h	.00076
2	1250	$1870h^2$	.295h	.00350
3	1500	$2666h^2$	.336h	.00667
4	1750	$3468h^2$	.369h	.01000
5	2000	$4384h^2$	.398h	.01380

c = allowed compression per □" on Concrete, in lbs.
M - Bending moment in inch pounds.
hx = distance from Neutral axis to Compression face of beam
p = ratio of area of steel to concrete

Vertical Reactions due to Vertical Loads

Load at		1	2	3	4	5
Live Load	P_1	4972	5250	5166	4600	3636
	P_2	1432	610	1424	2500	3636
Dead Load	P_1	17652	11520	7116	4448	2950
	P_2	508	1340	1964	2440	2950

Note.-Vertical Reactions due to Horizontal Loads neglected as unimportant.—

Table Vertical Dead Load. 50' Arch.

Panel Point	Earth Area	Concrete Area	Wt. of Earth	Wt. of Concr.	Total	X Factor of Safety 4
5	5	7.5	500	975	1475	5900
4	6.5	8.25	650	1072	1722	6888
3	11.0	9.00	1100	1170	2270	9080
2	18.5	10.5	1850	1365	3215	12860
1	28.5	13.0	2850	1690	4540	18160

Table Load on 50' Arch.

Panel Point	1	2	3	4	5
Live Load-Vertical	5116	5860	6590	7100	7270
" " Horizont.	852	780	548	472	0
Dead Load-Vertical	18160	12860	9080	6888	5900
" " -Horizont.	1900	986	370	174	0

Table Horizontal Components of the Reactions 50 Foot Arch.

Vertical Loads.

Panel Point —	1	2	3	4	5
Live Load —	970	3516	6810	9584	10648
Dead Load —	3444	7716	9398	9398	8642

Horizontal Load.

		1	2	3	4	5
Live L.	H_1 =	− 762	− 554	− 306	− 270	0
" "	H_2 =	+ 90	+ 226	+ 242	+ 202	0
Dead L.	H_1 =	−1700	− 702	− 210	− 88	0
" "	H_2 =	+ 200	− 290	+ 160	+ 86	0

Note:– Subscripts of H_1 & H_2 for panels 6, 7, 8, 9 are reversed.

Table of Moments – 50 Foot Arch.

Vertical Dead Loads

Load at ↓ / Mom at →	0	1	2	3	4	5	C.W.
9	+ 119840	+ 32684	– 27236	– 45364	– 70810	– 54468	453960
8	+ 346900	+ 51724	– 65580	– 135036	– 142772	– 92590	321500
7	+ 238760	+ 505720	– 84440	– 147090	– 136200	– 54476	227000
6	+ 198[illegible]00	+ 22726	– 82650	– 115716	– 76450	+ 33060	172200
5	+ 109740	– 10620	– 65490	– 54868	+ 21240	+ 166380	147500
4	0	– 53726	– 35128	+ 53726	+ 214900	+ 33060	172200
3	– 198840	– 98060	+ 76270	+ 324160	+ 58060	– 54480	227100
2	– 493840	– 69444	+ 412820	+ 185180	+ 15432	– 92560	321500
1	– 659120	+ 277800	+ 152520	+ 159920	– 10894	– 54470	453960
Total +	+ 933600	+ 900656	+ 641610	+ 622986	+ 345632	+ 232500	
–	– 1351800	– 231850	– 360524	– 518074	– 437126	– 403064	
Net M	– 418200	+ 668806	+ 281086	+ 104912	– 87494	– 170564	

Horizontal Dead Loads.

Load at ↓ / M at →	0	1	2	3	4	5
9	+ 7300	+ 1976	– 1612	– 3800	– 4408	– 3344
8	+ 360	+ 2290	– 2448	– 4976	– 5370	– 3554
7	+ 4610	+ 998	– 1380	– 2554	– 2494	– 1234
6	+ 2194	+ 388	– 750	– 1236	– 1054	– 236
5	0	0	0	0	0	0
4	– 2310	– 402	+ 932	+ 1374	+ 1250	– 236
3	– 6488	– 1056	+ 2760	+ 4960	+ 1262	– 1234
2	– 22348	– 918	+ 15084	+ 6712	+ 474	– 3554
1	– 40190	+ 16410	+ 9120	+ 3344	– 760	– 3344
Total +	+ 14464	+ 22062	+ 25796	+ 16390	+ 2986	+ 0
–	– 72330	– 2406	– 6250	– 12866	– 14086	– 16728
Net M	– 57866	+ 19656	+ 19546	+ 3924	– 11100	– 16728

Table of Moments - 50 Foot Arch

Vertical Live Loads.

M at → / Load at ↓	0	1	2	3	4	5	CW
9	+ 33766	+ 9206	- 7674	- 18416	- 19950	- 15346	127900
8	+ 112500	+ 28126	- 29882	- 61526	- 65040	- 42186	146500
7	+ 197740	+ 366940	- 61264	- 106720	- 96818	- 39524	164740
6	+ 204460	+ 23426	- 85190	- 119270	- 78602	+ 34076	177480
5	+ 154840	- 13050	- 80476	- 67440	+ 26580	+ 20440	181240
4	0	- 55760	- 36206	+ 53760	+ 221500	+ 34616	177480
3	- 44310	- 71164	+ 55350	+ 235240	+ 71164	- 39536	164740
2	- 225040	- 31644	+ 188092	+ 84390	+ 7032	- 42190	146500
1	- 185700	+ 78270	+ 42978	+ 168820	- 3070	- 15350	127900

Horizontal Live Loads

M at → / Load at ↓	0	1	2	3	4	5
9	+ 3272	+ 886	- 756	- 1704	- 1978	- 1500
8	+ 7248	+ 1810	- 1936	- 3536	- 4246	- 2812
7	+ 6896	+ 1422	- 2064	- 3820	- 3732	- 1844
6	+ 5976	+ 1056	- 2042	- 3366	- 2874	- 642
5	0	0	0	0	0	0
4	- 6264	- 1056	+ 2270	+ 3746	+ 3806	- 642
3	- 9684	- 1518	+ 536	+ 7424	+ 1864	- 1844
2	- 18384	- 7456	+ 11930	+ 5308	+ 394	- 2814
1	- 18270	+ 7360	+ 4090	+ 1500	- 340	- 1500

Table of Shear -
Between panel points

Vertical	0	1	2	3	4	5
Dead Load		49338	31776	18918	9838	2950
Vertical Max Live "		29530	24616	19368	14202	9602
Horizontal Dead		0	0	0	0	0
" Live		0	24	108	314	626
Temperature		918	714	510	306	102
Total Vert. Shear		80446	57134	38904	24660	13260
Shear perp. to ℄ Rib		68360	50880	36220	23900	13160
Required Sect. Area		228 □"	169.5 □"	117.5 □"	79.8 □"	43.8 □"

[redacted] Horizontal Thrust Due to Live Loads Giving Maximum Moments. 50 Foot Arch.

Section at →	5	4	3	2	1	0
Vertical Live Load	+ 22592	+ 21850	+ 20880	+ 11296	+ 21850	+ 20880
Horizont. " "	+ 736	+ 276	− 3524	− 2544	0	− 1892
Vertical Dead Load	+ 38492	+ 38492	+ 19246	+ 19246	+ 19246	+ 19246
Horizont. " "	+ 1460	+ 1268	+ 916	− 70	− 1970	− 1970
Temperature	+ 1602	+ 1602	+ 1602	+ 1602	+ 1602	+ 1602
Total	+ 64882	+ 63508	+ 58364	+ 48776	+ 59974	+ 57112
Factor	1.0	.99	.97	.93	.89	.85
Resolved Component	+ 64882	+ 62800	+ 56500	+ 45300	+ 53400	+ 48560

Thrust Due to Live Loads Giving Minimum Moments.

Section at →	5	4	3	2	1	0
Vertical Live Load	− Mom only	+ 21850	31528	− Mom practically = 0	+ Mom only	31528
Horizontal " "		852	760			1160
Vertical Dead Load		38492	38492			38492
Horizontal " "		1286	916			− 1970
Temperature		1602	1602			1602
Total		64082	73298			70412
Factor		.99	.97			.85
Resolved Component		63400	72780			59900

Note:- Factor = cosine of Angle between Horizontal and Tangent to Parabola at that point.

…Bending Moments - 50 Foot Arch Rib.

0	1		2		3		4		5	
…8-9	1-6-7-8-9		1-2-3		1-2-3-4		2-3-4-5		4-5-6	
…3-4	2-3-4-5		4-5-6-7-8-9		5-6-7-8-9		1-6-7-8-9		1-2-3-7-8-9.	
26411	+ 12606	− 10172	+ 16556	− 4522	+ 17960	− 12624	+ 6064	− 13172	− 1284	− 12314
418200	+668806	+668806	+281086	+281086	+104912	+104912	− 87494	− 87494	−170564	− 170564
57866	+ 19656	+ 19656	+ 19546	+ 19546	+ 3824	+ 3824	− 11100	− 11100	− 16728	− 16728
103400	+ 47500	− 47500	+ 4130	− 4130	+ 26850	− 26850	+ 45400	− 45400	+ 51700	− 51700
555050	+505970	−169618	+286416	−300686	+542210	−373372	+326676	−265682	+ 89132	− 194136
187338	+1264538	+460972	+607734	− 8706	+695756	−302310	+279546	−422848	− 47724	− 445442
48560	53400		45300		56500	72780	63400	62800		64882
11.65"		—		—		7.04"		6.94"		8.03"
2.21"		—		—		0.84"		1.32"		1.14"
	11.92"		9.10"		11.25"		9.32"		—	
	2.270"		1.68"		1.62"		0.42"		—	

Tabulation of Max. Bending Moments - 50 Foot Arch Rib.

Panel Point	0		1		2		3		4		5	
Live Load Vert. applied at. Pos. M.	5-6-7-8-9		1-6-7-8-9		1-2-3.		1-2-3-4		2-3-4-5		4-5-6	
Do Neg. M.	1-2-3-4.		2-3-4-5		4-5-6-7-8-9.		5-6-7-8-9.		1-6-7-8-9		1-2-3-7-8-9.	
L.L. Horizontal.	+ 23592	- 26411	+ 12605	- 10172	+ 16556	- 4522	+ 17960	- 12824	+ 6064	- 13172	- 1284	- 12314
D.L. Vertical	- 418200	- 418200	+658806	+658806	+281086	+281086	+ 104912	+ 104912	- 87494	- 87494	- 170564	- 170564
D.L. Horizontal	- 57866	- 57866	+ 19656	+ 19656	+ 19546	+ 19546	+ 3824	+ 3824	- 11100	- 11100	- 16728	- 16728
Temperature	+ 103400	- 103400	+ 47500	- 47500	+ 4130	- 4130	+ 26850	- 26850	+ 45400	- 45400	+ 51700	- 51700
L.L. Vertical	+675506	- 555050	+505570	- 169618	+286416	- 300586	+542210	- 373372	+326676	- 265682	+ 89132	- 194136
Total Moment	+324030	- 1167336	+1254538	+460972	+607734	- 8706	+695756	- 302310	+279546	- 422648	- 47724	- 445442
Thrust	59300	48560	53400		45300		56500	72780	63400	62800		64882
Distance below top of Section of Neutral Axis		11.65"		—		—		7.24"		6.94"		8.03"
Amount of Steel above Neutral Axis		2.21□"		—		—		0.64□"		1.32□"		1.14□"
Distance above bottom of Section of Neut. Axis	—		11.92"		9.10"		11.25"		9.32"		—	
Amount of Steel below Neutral Axis	—		2.27□"		1.68□"		1.62□"		0.42□"		—	

Table 6

Table of Ordinates to Parabola - 40 Foot Span.

Panel Point	0	1	2	3	4	5
Ordinate	0	2.25	4.00	5.25	6.00	6.25

Table of Dead Loads 40 Ft Span.

Panel Point	Earth Area Cu ft	Concrete Area Cu ft	Wt of Earth	Wt of Concrete	Total with Safety Factor of 4
0					
1	26.0	10.0	2600	1300	15600
2	20.0	8.0	2000	1010	12040
3	15.0	6.0	1500	780	9120
4	12.4	5.5	1240	650	7560
5	12.0	5.0	1200	650	7400

Table of Loads - 40 Foot Span.

Panel Point	0	1	2	3	4	5
Vertical DL	0	15600	12040	9120	7560	7400
" LL	0	3890	4260	4640	4880	4925
Horizontal DL	0	2600	1505	760	315	0
" LL	0	648	533	386	200	0

Table of Shear - 40 Foot Span

Between panel points

	0		1		2		3		4		5
Vert DL.		48000		32000		20360		11240		3680	
" L.L.		20133		16352		12532		8892		5732	
Hor DL. Negligible											
" L.L. do											
Temperature do											
Total Vert Shear		68133		48752		32892		20132		9412	
{Resolved " perp. to ℄ Rib		60000		44000				19500		9412	
Required Sect! Area		208□"		150□"				65□"		38□"	

Load at		1	2	3	4	5
Live Load	P_1	3780	3820	3640	3160	2462
	P_2	110	440	1000	1720	2462
Dead Load	P_1	15100	10800	7150	4900	3700
	P_2	500	1240	1970	2660	3700

Table of Moments - 40 Foot Arch

Vertical D.L.

Mom at / Load at	0	1	3	5	CW
9	+ 82500	+ 22500	- 44900	- 37400	312000
8	+ 185000	+ 46250	- 101200	- 69500	240800
7	+ 208000	+ 416000	- 118250	- 43800	182400
6	+ 174500	+ 19980	- 101800	+ 29100	151200
5	+ 110000	- 10670	- 55100	+ 167000	148000
4	0	- 47200	+ 47200	+ 29100	151200
3	- 159900	- 79000	+ 260500	- 43800	182400
2	- 370000	- 52050	+ 138800	- 69500	240800
1	- 454000	+ 191000	+ 41200	- 37400	312000
Total +M	+ 760000	+ 695730	+ 487700	+ 225200	
-M	- 983900	- 188920	- 421250	- 301400	
Net M.	- 223900	+ 506810	+ 66450	- 76200	

Horizontal D.L.

9	+ 7800	+ 2200	- 4060	- 3580
8	+ 10900	+ 2730	- 5920	- 4230
7	+ 7450	+ 1630	- 4140	- 1950
6	+ 3130	+ 560	- 1758	- 337
5	—	—	—	—
4	- 3290	- 570	+ 1950	- 337
3	- 10500	- 1710	+ 8030	- 1950
2	- 26600	- 1130	+ 8000	- 4230
1	- 44000	+ 17550	+ 3580	- 3580
Net M.	- 55110	+ 21260	+ 5690	- 20194

Vertical Live Loads.

9	+ 20400	+ 56000	- 11200	- 9320	77800
8	+ 65500	+ 16400	- 35800	- 24600	85200
7	+ 106000	+ 207000	- 60000	- 22200	92800
6	+ 112500	+ 12900	- 65500	+ 18750	97600
5	+ 73400	- 7100	- 36700	+ 111000	98500
4	0	- 30500	+ 30500	+ 18750	97600
3	- 81200	- 40000	+ 132500	- 22200	92800
2	- 131000	- 18400	+ 49100	- 24600	85200
1	- 113000	+ 47500	+ 10300	- 9320	77800

Horizontal Components of the Reactions
40 Foot Arch.

Load at →			1	2	3	4	5
Vertical Loads	Live		760	2620	4920	6745	7400
	Dead		3040	7420	9650	10450	11100
Horizontal Loads.	Live	H_1	578	389	225	102	0
		H_2	70	144	161	98	0
	Dead	H_1	2320	1070	443	161	0
		H_2	280	435	317	154	0
Temperature			1400	1400	1400	1400	1400

Thrust in 40 Foot Arch.

		0	1	3	5
Vertical Live Load	+ Moment	29200	18090	15050	20900
	− Moment	15050	21690	22450	16600
Horizontal Live Load	+ Moment	+473	+543	−1179	−319
	− Moment	−1284	−716	+473	−2384
Vertical Dead Load		72220	72220	72220	72220
Horizontal " "		−2808	−208	+2057	+2372
Temperature		1400	1400	1400	1400
Total for + Moment		100485	92045	89548	93573
" " − "		84568	94386	98600	90206
Resolved Component for + Mom.		85700	82900	88600	93573
" " " − "		71900	84900	97600	90206

Table of Bending Moments - 40 Foot Arch.
Horizontal Live Loads.

M at → / Load at ↓	0	1	3	5
9	+ 1940	+ 520	- 1012	- 880
8	+ 3860	+ 965	- 2100	- 1500
7	+ 3790	+ 820	- 2100	- 1010
6	+ 1980	+ 350	- 1110	- 210
5	—	—	—	—
4	- 2080	- 360	+ 1240	- 210
3	- 5330	- 870	+ 4070	- 1010
2	- 9400	- 400	+ 2820	- 1500
1	- 11000	+ 4320	+ 880	- 880

Tabulation of Moments, Thrusts, Sections, Etc. For 40 Foot Arch

Panel Point	0		1		3		5	
Live Loads applied - Max. +M	5-6-7-8-9		1-6-7-8-9		1-2-3-4		4-5-6	
" " " " " -M		1-2-3		2-3-4		5-6-7-8-9		1-2-3-7-8-9
Mom. due to Vert. Live Load	+ 377800	− 325200	+ 333800	− 96000	+ 222400	− 209200	+ 148500	− 112240
" " " Hor. " "	− 11570	− 27810	+ 6975	− 1630	+ 9010	− 6322	− 420	− 6780
" " " Vert. Dead Load	− 223900	− 223900	+ 506810	+ 506810	+ 66450	+ 66450	− 76200	− 76200
" " " Hor. " "	− 55110	− 55110	+ 21260	+ 21260	+ 5690	+ 5690	− 20194	− 20194
" " " Temperature	+ 70600	− 70600	+ 32400	− 32400	+ 18350	− 18350	+ 35300	− 35300
Total Moment	+ 180960	− 702620	+ 907245	+ 398040	+ 321900	− 161732	+ 86986	− 250714
Thrust	85700	71900	82900		88600	97600	93570	90206
Distance of Neutral Axis below top of Section →		13.3"		—		7.49"		7.6"
Amount of Steel		2.09□"		—		0.34□"		0.55□"
Distance of Neutral Axis above bottom of Section →	5.6"		11.4"		8.34"		6.4"	
Amount of Steel	0.36□"		1.96□"		0.68□"		0.079□"	